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Effects of Zonal Fields on Energetic-Particle Excitations of Reversed Shear Alfvén Eigenmodes: Simulation and Theory†

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(I) Introduction

(II) GTC Simulations

(III) Theoretical Analyses

- Beat-driven zonal fields
- EP instability drive and phase-space zonal structures

(IV) Summary & Discussions

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(I) Introduction

Energetic-Particle

◎ EP ⇒ RSAE (AE's) ⇒ Zonal e.m. fields (ZFs) and $\boxed{\text{Zonal}}$ $\boxed{\text{phase-space}}$ structures (PSZS) ⇒ suppress RSAE ⇒ NL saturation

Lower level

Well established (mainly via GK simulations)

?? How do ZFs / PSZS suppress RSAE?

No definitive / quantitative analysis

◎ Two possible routes:

(i) Via thermal plasmas ⇒ modification in mode structures, NL frequency shift, etc. ⇒ enhanced damping? ⇒ $\gamma_{NL} = \gamma_{NL}(ZFs)$??

* (ii) via EPs ⇒ ZFs shearing ⇒ reduces EP drive ??

◎ Focus of the work: Investige route (ii) via

Combining simulations & analytical theory

{Computer experiments to gain/improve analytical theory/physics insights}

⊙ Highlights of results (Simulation & Theory):

⇒ ZFs do NOT suppress EP's drive of RSAE !!

⇒ ZFs enhances EP's instability drive via the destabliting ZFs-induced PSZS.

⇒ ZFs suppress/saturate RSAE via NL mechanisms in thermal plasmas

: How ?? under investigation

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(II) GTC Simulations

⊙ $$\boxed{f = \left(\frac{q}{m}\right)\frac{\partial f_0}{\partial \varepsilon}\left(1 - e^{-\underset{\sim}{\rho}\cdot\underset{\sim}{\nabla}} J_0\right)\delta\phi + e^{-\underset{\sim}{\rho}\cdot\underset{\sim}{\nabla}} f_g}$$

polarization

⊙ f_g : gyro-center distribution function

$$\boxed{\left(\mathcal{L}_g + \underbrace{\delta\mathcal{L}_x + \delta\mathcal{L}_\varepsilon}_{\delta\mathcal{L}}\right) f_g(\varepsilon, \mu, \underset{\sim}{x}, t) = 0}$$

g-c NLGKE

⊙ $$\boxed{\mathcal{L}_g = \partial_t + v_\parallel \underset{\sim}{b}_0 \cdot \underset{\sim}{\nabla} + \underset{\sim}{v}_d \cdot \underset{\sim}{\nabla}}$$

∘ $\underset{\sim}{v}_d$: ∇B_0 + Curvature drift

⊙ $$\boxed{\delta\mathcal{L}_x = \langle \delta \underset{\sim}{u}_g \rangle \cdot \underset{\sim}{\nabla}}$$

∘ $$\langle \delta \underset{\sim}{u}_g \rangle = \frac{c}{B_0} \underset{\sim}{b}_0 \times \left\langle \left(\delta\phi - \frac{v_\parallel \delta A_\parallel}{c}\right)_g \right\rangle = \langle \delta \underset{\sim}{U}_E \rangle + v_\parallel \langle \delta \underset{\sim}{B}_\perp \rangle / B_0$$

⊙ $$\boxed{\delta\mathcal{L}_\varepsilon = \delta\dot{\varepsilon}\,\frac{\partial}{\partial\varepsilon}}$$

• $$\delta\dot{\varepsilon} = \left(\frac{q}{m}\right)\left[v_\parallel \left(\underset{\sim}{b}_0 + \frac{\langle \delta \underset{\sim}{B}_\perp \rangle}{B_0}\right) \cdot \langle \delta \underset{\sim}{E} \rangle + \underset{\sim}{v}_d \cdot \langle \delta \underset{\sim}{E} \rangle \right]$$

◎ Keeping only a single-n_0 RSAE + the zonal components of fluctuations

$$\Rightarrow \quad \delta\mathcal{L}_X + \delta\mathcal{L}_Z \stackrel{\Delta}{=} \boxed{\delta\mathcal{L} = \delta\mathcal{L}_0 + \delta\mathcal{L}_Z}$$

$$\Rightarrow \quad \boxed{f_g = F_{g0} + \delta F_g}$$

$$\Rightarrow \quad \boxed{\left[\mathcal{L}_g + \delta\mathcal{L}_0 + \delta\mathcal{L}_Z\right]\delta F_g = -\left[\delta\mathcal{L}_0 + \delta\mathcal{L}_Z\right] F_{g0}} \quad (*)$$

◎ Three cases of study on EP physics

- Case A : No ZFs $\Rightarrow$ $\boxed{\delta\mathcal{L}_Z = 0}$ in $(*)$ EP

$$\boxed{\left[\mathcal{L}_g + \delta\mathcal{L}_0\right]\delta F_g = -\delta\mathcal{L}_0 F_{g0}}$$

- Case B : Full ZFs $\Rightarrow$ Eq. $(*)$

- Case C : Partial ZFs $\Rightarrow$ $\delta\mathcal{L}_Z = 0$ in the g.c. Propator (LHS of $(*)$)

$$\boxed{\left[\mathcal{L}_g + \delta\mathcal{L}_0\right]\delta F_g = -\left[\delta\mathcal{L}_0 + \delta\mathcal{L}_Z\right] F_{g0}}$$

suppress shearing

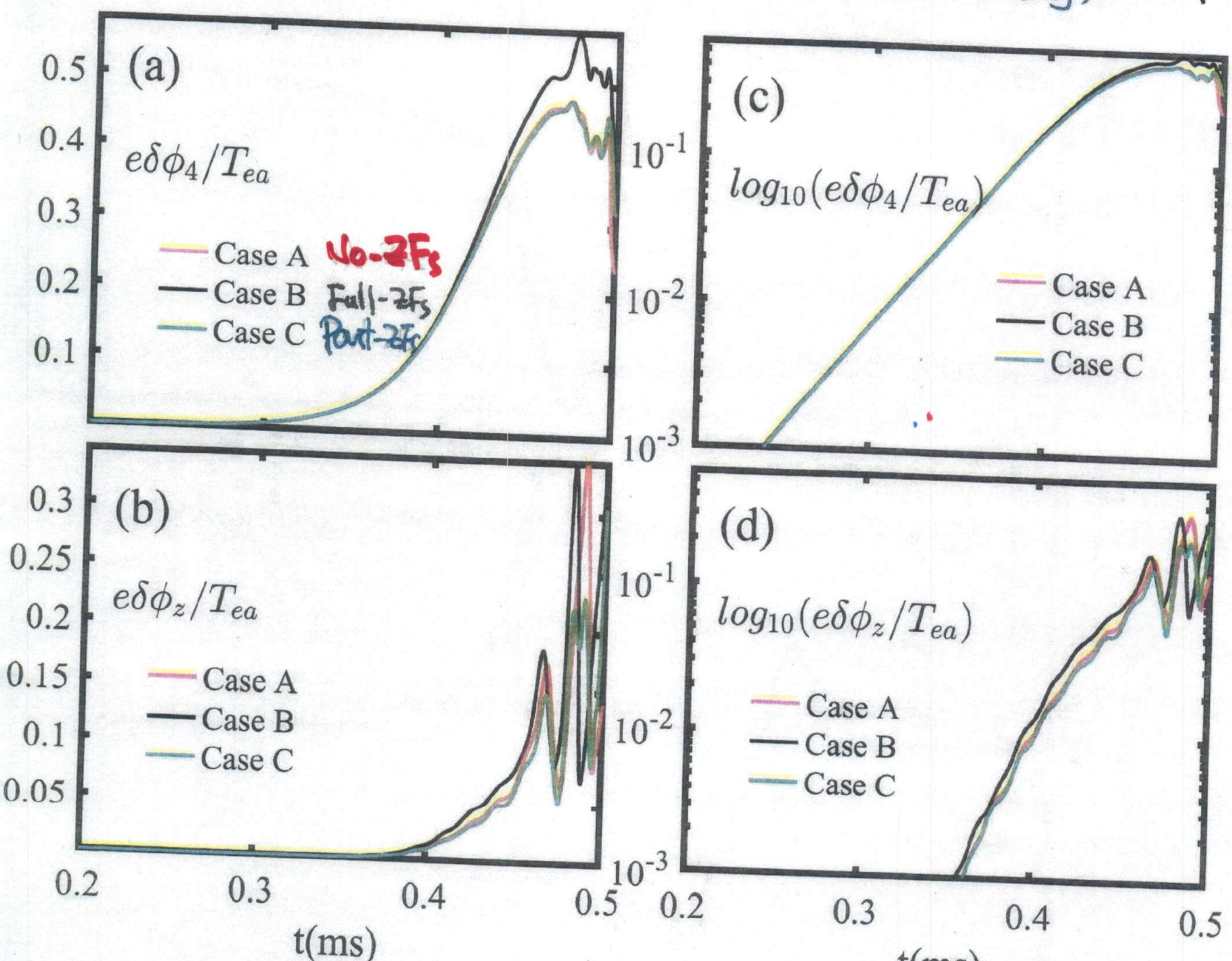
(II) GTC Simulations of RSAE : ($\delta\phi_4$, $\delta\phi_z$)

Case A : No ZFs on EP

DIII-D case

Case B : Full ZFs on EP

Case C : Partial ZFs (No zonal drift/shearing) on EP

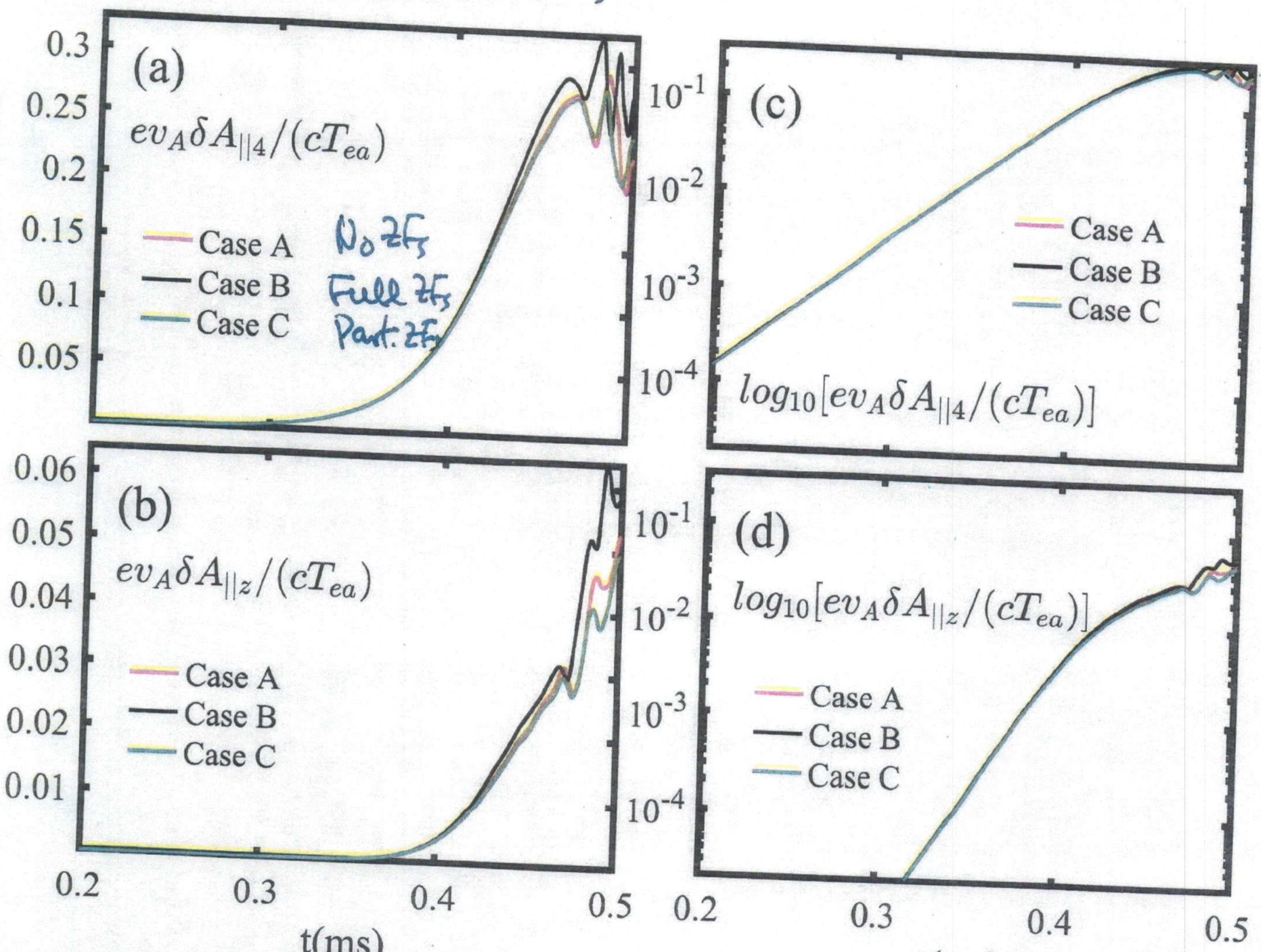


GTC Simulations of RSAE: ($\delta A_{\parallel 4}$, $\delta A_{\parallel z}$)

Case A: No-ZFs on EP

Case B: Full-ZFs on EP

Case C: Partial-ZFs on EP



(III) Theoretical Analyses

(III.a) Beat-driven ZFs by RSAE

⊙ Governing eqns

• NLGKE ($\partial F_0/\partial\mu = 0$) (F-C, 1982) ⇒ g-C NLGKE + $O(\rho/R)$ parallel NL

⊙ $$\boxed{f = F_0 + \left(\frac{q}{m}\right)\frac{\partial F_0}{\partial \varepsilon}\delta\phi + e^{-\rho\cdot\nabla}\delta g}$$

⊙ $$(\mathcal{L}_g + \delta\mathcal{L}_x)\,\delta g = -\frac{q}{m}\frac{\partial F_0}{\partial \varepsilon}\frac{\partial}{\partial t}\left\langle\left(\delta\phi - \frac{v_\parallel \delta A_\parallel}{c}\right)\right\rangle_\alpha - \delta\mathcal{L}_x F_0 \equiv -\frac{q}{m}QF_0\left\langle\delta\phi - \frac{v_\parallel\delta A_\parallel}{c}\right\rangle_\alpha$$

⊙ $QF_0 = [(\partial F_0/\partial\varepsilon)(i\partial_t) + (\underset{\sim}{k}\times \underset{\sim}{b}_0/\Omega)\cdot\nabla F_0]$

• Quasi-neutrality condition

$$\boxed{\frac{N_0 e^2}{T_e}(1+\tau)\delta\phi = \sum_{j=e,i} e_j \langle J_0 \delta g_j\rangle_v}$$

• Parallel Ampere's Law

$$\boxed{\nabla_\perp^2 \delta A_\parallel = \frac{4\pi}{c}\delta J_\parallel = \frac{4\pi}{c}\sum_j e_j\langle J_0 v_\parallel \delta g_j\rangle_v}$$

⊙ Calculate

G-C NLGKE ⇒

$$\begin{cases} \bullet\ \delta g_{z,j} = \delta g_{z,j}^{(1)} + \delta g_{z,j}^{(2)} \\ \bullet\ \delta g_{z,j}^{(1)} = \delta g_{z,j}^{(1)}[\delta\phi_z, \delta A_{\parallel z}] \\ \bullet\ \delta g_{z,j}^{(2)} = \delta g_{z,j}^{(2)}[\delta\phi_0, \delta A_{\parallel 0}] \end{cases}$$

- $$\boxed{\begin{pmatrix}\delta\phi_0\\ \delta A_{110}\end{pmatrix} = e^{-i\omega_{0r}t + in_0\xi}\sum_m \begin{pmatrix}\Phi_m(r,t)\\ A_m(r,t)\end{pmatrix} e^{-im\theta} + c.c.}$$
  - RSAE fluctuations
  - $\underline{\delta E_{110} \simeq 0} \Rightarrow \dfrac{\omega_0\,\delta A_{110}}{c\,k_{110}} = \delta\phi_0$

- <u>Beat-driven ZFs</u>
  - $$\boxed{\delta\phi_z = \frac{c}{B_0\omega_{0r}^2}(1+c_0\eta_i)\frac{\partial}{\partial r}\left[\sum_m\left(\frac{n_0 q}{r}\right)\omega_{*in}|\Phi_m|^2\right]}$$
  - $\underline{C_0 \simeq 1}$ for $|k_z\rho_{bi}|^2 \ll 1$

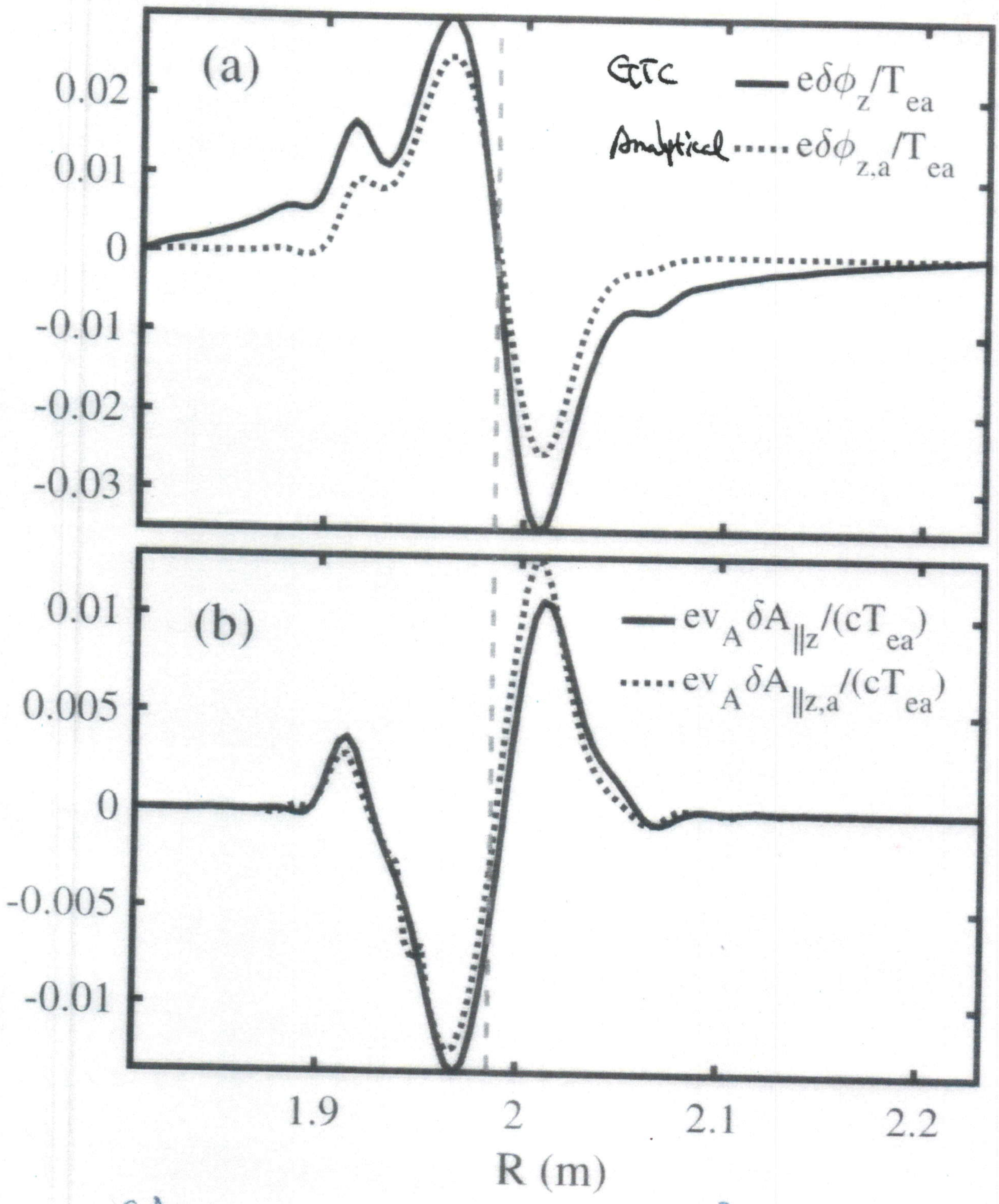
- $$\boxed{\frac{\delta A_z}{c} = \frac{c}{B_0\omega_{0r}^2}\frac{\partial}{\partial r}\left[k_{110}\left|\frac{\omega_0\,\delta A_{11}}{c\,k_{11}}\right|_0^2\right]}$$

- $|\delta\phi_z|, |\delta A_z| \propto \partial_r|\delta\phi_0|^2, \partial_r|\delta A_{110}|^2$

  $\Rightarrow$ <u>ponderomotive</u> force of RSAE

- Valid for high-frequency AE's : <u>TAE etc.</u>

$$\delta\phi_z = \frac{c}{B_0}\frac{1}{\omega_{0r}^2}(1+\eta_i)\,\partial_r\left[k_{\theta 0}\,\omega_{*in}\,|\delta\phi_0|^2\right]$$



$$\frac{\delta A_z}{c} = \frac{c}{B_0\,\omega_{0r}^2}\,\frac{\partial}{\partial r}\left[k_{\theta 0}\,k_{\parallel 0}\left|\frac{\omega_0\,\delta A_\parallel}{c\,k_\parallel}\right|_0^2\right]$$

(III.b) AE stability + phase-space zonal structure

◉ General fishbone-like dispersion relation (GFLDR)

$\Rightarrow$ EP contribution to the RSAE (AE) drive [Z&C, PoP, 2014 a, b]

$$\boxed{\mathrm{Im}\,\delta W_{k0} = e_E\, \mathrm{Im} \int d^3\underset{\sim}{x}\, \delta\phi_0^* \left\langle (J_0\omega_z + \omega_d J_0)\delta K_0 \right\rangle_v}$$

w-p resonance

- $\omega_z = -i\langle \delta \underset{\sim}{u}_g \rangle_z \cdot \underset{\sim}{\nabla}_\perp$, • $\omega_d = -i\underset{\sim}{v}_d \cdot \underset{\sim}{\nabla}_\perp$

○ $$\boxed{(\mathcal{L}_{g0} + i\omega_z)\delta K_0 = i\left(\frac{e}{m}\right) J_0 \frac{(\omega_d + \omega_z)}{\omega_{0r}} \delta\phi_0\, Q(F_0 + \delta g_z)}$$

- $QF_0 = \left(i\frac{\partial F_0}{\partial \mathcal{E}}\frac{\partial}{\partial t} + \hat{\omega}_* F_0\right)$

$$\boxed{\bullet\ \mathcal{L}_g \delta g_z = -[\delta\mathcal{L}_0 \times \delta g_0]_z - \left(\frac{e}{m}\right)\left[\frac{\partial F_{g0}}{\partial \mathcal{E}} \times \partial_t\left(\delta\phi - \frac{v_\parallel \delta A_\parallel}{c}\right)\right]_z}$$

- δK_0 : compressional component of δf_0 containing w-p resonance [C+H, JGR, 1991]

$\Rightarrow$ $\boxed{\mathrm{Im}\,\delta W_{k0} > 0 \Rightarrow \text{instability drive}}$

○ $|\omega_{0r}| \ll |\omega_{*iE}|$

$\Rightarrow$ $Q \simeq \hat{\omega}_* = (\underset{\sim}{k} \times \underset{\sim}{b}_0/\Omega)\cdot\hat{r}\,\partial/\partial r$

$\Rightarrow$ $QF_0|_{r_m} > 0$ for $(\partial F_0/\partial r)_{r_m} < 0$: linear instability drive ($|\delta\phi_0|$ peaks at $q(r_m) = q_{min}$)

?? $(\partial \delta g_z/\partial r)_{r_m}$??

R.K.

◎ Connection between simulation δF_g and analytical δg :

analytical • $$\boxed{f = F_0 + \left(\frac{e}{m}\right)\frac{\partial F_0}{\partial \varepsilon} + e^{-\rho\cdot\nabla}\delta g}$$

Simulation • $$f = \left(\frac{e}{m}\right)\frac{\partial F_0}{\partial \varepsilon}\left(1 - e^{-\rho\cdot\nabla} J_0\right)\delta\phi + e^{-\rho\cdot\nabla} f_g$$

• $f_g = F_{g0} + \delta F_g$

$$\Rightarrow \boxed{\delta g = \delta F_g - \left(\frac{e}{m}\right)\frac{\partial F_{g0}}{\partial \varepsilon} J_0 \delta\phi}$$

(i) Case A : No ZFs in EP ($\delta\phi_z = 0 = \delta A_{\parallel z}$)

◎ $$\mathrm{Im}\,\delta W_{KOA} = \left(\frac{e^2}{m}\right)_E \left(\frac{\pi}{\omega_{0r}}\right)\int d^3x \left\langle J_0^2 \partial_{E0}^2 |\delta\phi_0|^2 \bar{\omega}_d^{-2} \times \delta(\bar{\omega}_d - \omega_0) Q(F_0 + \delta g_{ZA})_E \right\rangle_v$$

• Assuming trapped EPs • ∂_{E0} : finite banana-width ($k_r \rho_b$)

◎ $$\boxed{L_{g0}\delta g_{ZA} = -\left[\langle \delta U_g\rangle_0 \cdot \nabla \delta g_{0A}\right]_z}$$

• PSZS due to symmetry-breaking RSAE only ! ⇒ "clump-hole" PS structures!

○ O'Neil's single-wave model!

- $$\boxed{\delta g_{zA} \simeq \mathcal{J}_{zE}^2 \, g_{E0}^2 \left| \frac{c}{B_0} \frac{n_0 q}{r} \frac{\overline{\omega}_d}{\omega_{0r}} \right|^2 \frac{\partial}{\partial r} \left[\frac{|\delta\Phi_0|^2}{(\overline{\omega}_d - \omega_{0r})^2 + \gamma_z^2} \cdot \frac{\partial F_0}{\partial r} \right]}$$

$$\Rightarrow \quad \delta g_{zA} : \left(\begin{array}{lll} r > r_m \Rightarrow > 0 & \text{clump} \\ r < r_m \Rightarrow < 0 & \text{hole} \end{array} \right)$$

$$\Rightarrow \boxed{\frac{\partial}{\partial r} \delta g_{zA} > 0 \quad \text{at } r_m}$$

$\Rightarrow$ stabilizing

(ii) Case B : Full ZFs in EP dynamics

- $$\mathrm{Im}\,\delta W_{KDB} \simeq \left(\frac{e^2}{m}\right)_E \frac{\pi}{\omega_{0r}} \int d^3\underset{\sim}{x} \left\langle J_0^2 \mathcal{J}_{E0}^2 |\delta\Phi_0|^2 (\overline{\omega}_d + \omega_{zE})^2 \delta(\overline{\omega}_d + \omega_{zE} - \omega_0)\, Q\,(F_0 + \delta g_{zB})_E \right\rangle_v$$

- $$\boxed{\delta g_{zB} = \delta g_{zA} + \delta g_z^{(1)}}$$

- $$\boxed{\delta g_z^{(1)} \simeq -\left(\frac{e}{m}\frac{\partial F_0}{\partial \varepsilon}\right) J_z \mathcal{J}_{E3}^2 \, \delta\phi_z}$$

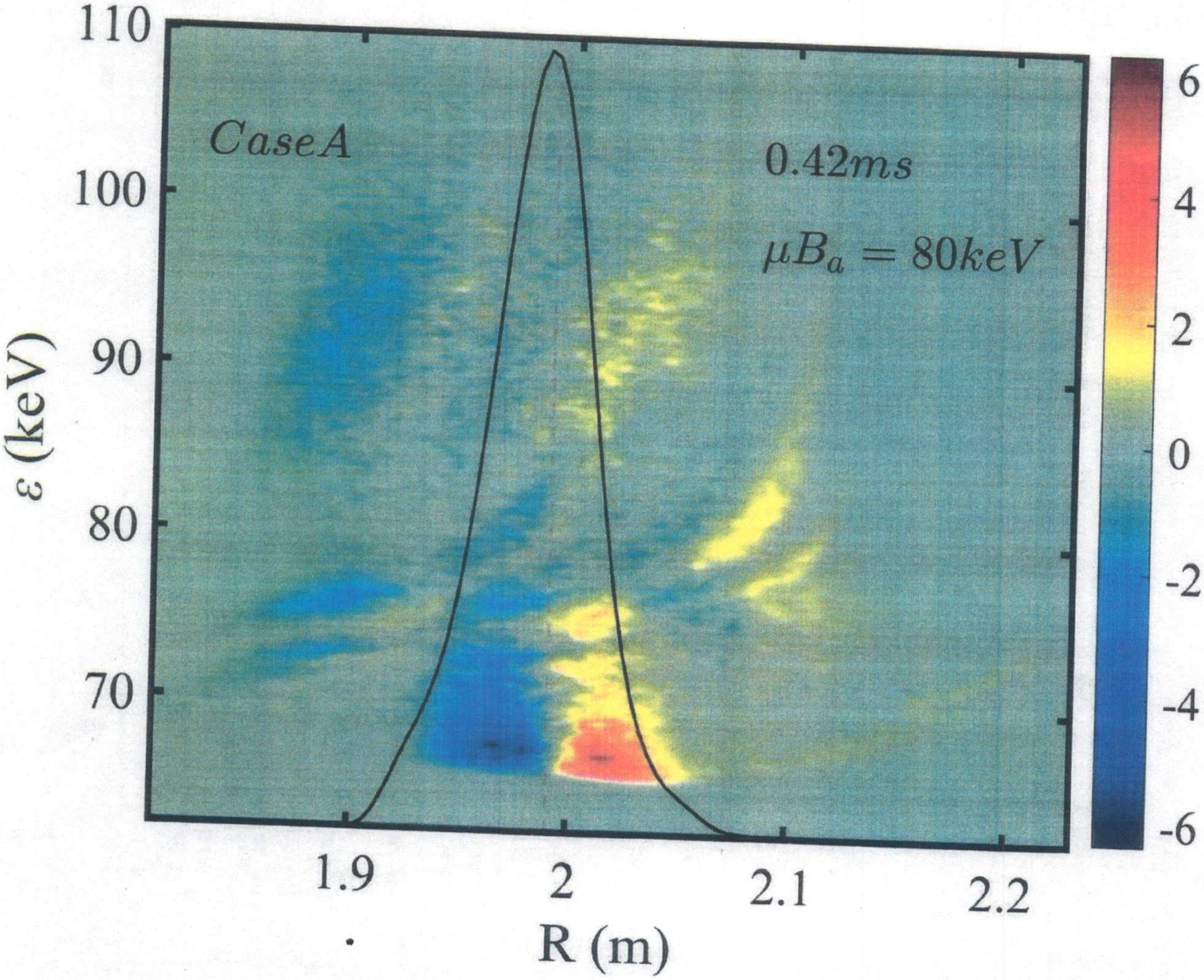
- $\omega_{zE} = \underset{\sim}{k_0} \cdot \overline{\langle \delta \underset{\sim}{u}_{gz} \rangle} \sim O(\gamma_L) \ll |\omega_0|, |\overline{\omega}_d|$

$\Rightarrow$ Negligible shift in W-P resonance

$\delta F_z(\mu, \varepsilon, R)$

Case A : No-ZFs in EP

hole-clump due to $(\delta\phi_x, \delta A_{\parallel x})$ only



hole clump

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$$\Rightarrow \left(\frac{\partial}{\partial r}\delta g^{(1)}_{zE}\right)\Big|_{r_m} \doteq -\left(\frac{e}{m}\frac{\partial F_0}{\partial \mathcal{E}}\right)\frac{c}{B_0}\frac{(1+C_0\eta_i)}{\omega_{0r}^2}k_{\theta 0}\,\omega_{*in}\frac{\partial^2}{\partial r^2}|\delta\phi_0|^2\Big|_{r_m} < 0.$$

$\Rightarrow$ destablizing !

(iii) Case (c): Partial ZFs in EP dynamics

$\Rightarrow$ turning off ZF shearing/drift term

∘ $$\mathrm{Im}\,\delta W_{KOC} = \left(\frac{e^2}{m}\right)_E\frac{\pi}{\omega_{0r}}\int d^3\underset{\sim}{x}\,\Big\langle J_0^2\,\partial_{E0}^2\,|\delta\phi_0|^2\,\bar{\omega}_d^2\,\delta(\bar{\omega}_d-\omega_0)\,Q\,(F_{g0}+\delta F_{gzC})\Big\rangle_v$$

∘ $$\boxed{\delta F_{gzC}} = \left(\frac{e}{m}\right)\frac{\partial F_0}{\partial\mathcal{E}}J_0\delta\phi_z + \delta g_{zB}$$

$$= \underbrace{\delta g_{zA} + }_{\delta g_{zB}}\Big\{\delta g_z^{(1)} + \left(\frac{e}{m}\right)\frac{\partial F_0}{\partial\mathcal{E}}J_0\delta\phi_z\Big\}$$

$$\Downarrow$$

$$\frac{\partial}{\partial r} > 0$$

∘ $$\{\dots\} = \left(\frac{e}{m}\right)_E\frac{\partial F_0}{\partial\mathcal{E}}J_z(1-\partial_{E0}^2)\delta\phi_z.$$

$\hookrightarrow$ stablizing w.r.t. Case B: Full ZFs

$\Rightarrow |1-\partial_{E0}^2| \ll 1$, $|k_z\rho_{bE}| \ll 1$

$\Rightarrow \frac{\partial}{\partial r}\{\dots\} < 0 \Rightarrow$ weakly stablizing w.r.t. Case (A)

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(IV) Summary & Discussions

- NL gyrokinetic simulation & theory
  ⇒ ZFs beat driven by RSAE in good agreement.

- ZFs ⇒ EP phase-space zonal structures ⇒ enhance the EP instability drive !!

- ZFs suppress RSAE via nonlinear physics of thermal plasmas
  ⇒ How ?? Under investigation.

- What about TAE ??

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Appendices

(I) In simulations

⊙ $$\boxed{(\mathcal{L}_g + \delta\mathcal{L}_0 + \delta\mathcal{L}_z)\,\delta F_g = -(\delta\mathcal{L}_0 + \delta\mathcal{L}_z)\,F_{g0}}$$

⊙ $$\boxed{\delta F_g = \delta F_{g0} + \delta F_{gz}} \qquad \boxed{\delta\mathcal{L}_k = \delta\mathcal{L}_{kX} + \delta\mathcal{L}_{k\mathcal{E}}}$$

$n_0 \Rightarrow$ $$\boxed{(\mathcal{L}_g + \delta\mathcal{L}_z)\,\delta F_{g0} = -\,\delta\mathcal{L}_0\,(F_{g0} + \delta F_{gz})}$$

$z \Rightarrow$ $$\boxed{\mathcal{L}_g\,\delta F_{gz} \simeq -\,\delta\mathcal{L}_z F_{g0} - [\delta\mathcal{L}_0\,\delta F_{g0}]_z}$$

⊙ $$\boxed{\delta g = \delta F_g - (e/m)(\partial F_{g0}/\partial\mathcal{E})\,J_0\,\delta\phi}$$

(II) Case A: $\delta\mathcal{L}_z = 0$ No ZFs $\Rightarrow \delta F_{gzA} = \delta g_{zA}$

⊙ $$\mathcal{L}_g\,\delta F_{g0A} = -\,\delta\mathcal{L}_0\,(F_{g0} + \delta g_{zA})$$

$\Rightarrow$ $$\boxed{\mathcal{L}_g\,\delta g_{0A} \simeq i\left(\frac{e}{m}\right) Q\,(F_{g0} + \delta g_{zA})\,J_0\left(\delta\phi - \frac{v_\parallel\,\delta A_\parallel}{c}\right)_0}$$

• $$Q = [\,i(\partial/\partial t)(\partial/\partial\mathcal{E}) + \hat{\omega}_*\,] \simeq \hat{\omega}_* = (\mathbf{k}\times\mathbf{b}_0/\Omega)\cdot\nabla$$

⊙ $$\boxed{\mathcal{L}_g\,\delta g_{zA} \simeq -\left[\langle\delta\mathbf{u}_g\rangle_0\cdot\nabla\,\delta g_{0A}\right]_z}$$

⊙ Extracting the compressional component

$$\delta g_{0A} = -\left(\frac{e}{m}\right)\left(\frac{Q}{\omega_{0r}}\right)(F_0 + \delta g_{zA})\,J_0\,\delta\psi_0 + \delta K_{0A}$$

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(III) Case B : $\delta \mathcal{L}_z \neq 0$ Full ZFs [F-C NLGKE]

$$\boxed{(\mathcal{L}_g + \delta \mathcal{L}_z)\, \delta g_{0B} = i\left(\frac{e}{m}\right) Q (F_{g0} + \delta g_{zB})\, J_0 \left(\delta\phi - \frac{v_\parallel \delta A_\parallel}{c}\right)_0}$$

$$\boxed{\delta g_{zB} = \delta g_{zA} + \delta g_z^{(1)}} \Leftarrow \mathcal{L}_g \delta g_{zB} = i\left(\frac{e}{m}\right) Q F_0 J_z \left(\delta\phi - \frac{v_\parallel \delta A_\parallel}{c}\right)_z - \left[\langle \delta \underset{\sim}{u}_g \rangle_0 \cdot \underset{\sim}{\nabla} \delta g_{0B} \right]_z$$

$$\boxed{\mathcal{L}_g \delta g_z^{(1)} = -\left(\frac{e}{m}\right) \frac{\partial F_{g0}}{\partial \varepsilon} \frac{\partial}{\partial t} J_z \left(\delta\phi - \frac{v_\parallel \delta A_\parallel}{c}\right)_z}$$

(IV) Case C : Partial ZFs ⇒ $\delta \mathcal{L}_z = 0$ in the g.c. propagator

$$\mathcal{L}_g \delta F_{g0C} = -\delta \mathcal{L}_0 (F_{g0} + \delta F_{gzC})$$

$$\Rightarrow \boxed{\mathcal{L}_g \delta g_{0C} = i\left(\frac{e}{m}\right) Q (F_{g0} + \delta F_{gzC})\, J_0 \left(\delta\phi - \frac{v_\parallel \delta A_\parallel}{c}\right)_0}$$

$$\boxed{\delta F_{gzC} = \delta g_{zC} + \left(\frac{e}{m}\right) \frac{\partial F_{g0}}{\partial \varepsilon} J_z \delta\phi}$$

$$\boxed{\delta g_{zC} = \delta g_{zB}}$$